Statistics

- Extract sensible information
- Investigate trends or patterns
- Find associations between variables

Health Data:

- Daily Exercise
- Plasma Cholesterol (mmol/L)
  - A) 60 + minutes
  - B) 31 - 60 minutes
  - C) 15 - 30 minutes
  - D) < 15 minutes
Statistics

- Extract sensible information
- Investigate trends or patterns
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Health Data:

<table>
<thead>
<tr>
<th>Daily Exercise</th>
<th>Plasma Cholesterol (mmol/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) 60 + minutes</td>
<td></td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
<tr>
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</table>
Analysis of Variance

Are the groups (A, B, C, D) actually different from one another in terms of the measured plasma cholesterol levels?
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\[ H_0 : \mu_A = \mu_B = \mu_C = \mu_D \ \quad (1) \]
\[ H_1 : \mu_i \neq \mu_j \text{ for at least one pair } (i, j) \ \quad (2) \]
1. Split the data into groups corresponding to the different levels of the variable *Exercise*.

2. Analyse the variances among groups and compare to variances within groups.
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Analysis of Variance

Data Modelling:

\[ y_{ij} = \mu + \tau_i + \epsilon_{ij} \]

\[ \begin{align*}
    i &= 1, 2, 3, 4 \\
    j &= 1, 2, 3, \ldots, n
\end{align*} \]
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Data Modelling:

\[ y_{ij} = \mu + \tau_i + \epsilon_{ij} \]

\[ \begin{cases} 
  i = 1, 2, 3, 4 \\
  j = 1, 2, 3, \ldots, n 
\end{cases} \]

- \( y_{ij} \) represents the ijth observation
- \( \mu \) represents the overall mean (i.e. the mean pooled across all levels)
- \( \tau_i \) is a unique parameter to each group level and is referred to as the \textit{treatment effect}. \( \tau_i \) represents the deviation from the overall mean resulting from the ith group level.
- \( \epsilon_{ij} \) is the random error of the experiment. The random error represents other sources of variability (eg. variability due to measurement errors or due to background noise.)
Analysis of Variance

Compare variances among groups to variances within groups with the F-Test:

\[
F_0 = \frac{SS_{Levels}}{a-1} = \frac{MS_{levels}}{MS_E} = \frac{\text{Variation among groups}}{\text{Variation within groups}}
\]
Analysis of Variance

Compare variances among groups to variances within groups with the F-Test:

\[ F_0 = \frac{SS_{Levels}}{a-1} \cdot \frac{SS_E}{N-a} = \frac{MS_{levels}}{MS_E} = \frac{\text{Variation among groups}}{\text{Variation within groups}} \]
The null hypothesis should be rejected if

\[ F_0 > F_{\alpha, a-1, N-a} \]
> setwd("C:/Users/Peter/Documents/Summer Research/Topic 1 R")
> data <- read.csv("heartdata.csv", header = TRUE, sep = ",")

> model.plasma <- aov(plasma.ch~ exercise, data = new.data) #Set up the One-Way ANOVA

> summary(model.plasma)

    Df Sum Sq  Mean Sq F value Pr(>F)    
exercise  3   86.0  28.7233  27.330 <2e-16 ***
Residuals 11730 12329.0  1.0513

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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Analysis of Variance
Assumption: The errors are normally and independently distributed random variables

$$\epsilon_{ij} \sim N(0, \sigma^2)$$
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\[ \epsilon_{ij} \sim \mathcal{N}(0, \sigma^2) \]
Problems?

\[ H_0 : \mu_A = \mu_B = \mu_C = \mu_D \] (3)
\[ H_1 : \mu_i \neq \mu_j \text{ for at least one pair } (i, j) \] (4)
Multiple Comparisons

\[ A \leftrightarrow B \quad B \leftrightarrow C \quad C \leftrightarrow D \]
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**Table:** All pairwise comparisons
Multiple Comparisons

$A \leftrightarrow B \quad B \leftrightarrow C \quad C \leftrightarrow D$

$A \leftrightarrow C \quad B \leftrightarrow D$

$A \leftrightarrow D$

**Table:** All pairwise comparisons

\[
\text{T-test:} \quad \frac{\bar{X}_1 - \bar{X}_2}{Sp \sqrt{\left( \frac{1}{N_1} + \frac{1}{N_2} \right)}}
\]
Multiple Comparisons

\[ T\text{-test: } \frac{\bar{X}_1 - \bar{X}_2}{Sp \sqrt{(1/N_1 + 1/N_2)}} \]
Multiple Comparisons

Type 1 error rate ($\alpha$): $\text{Pr(}\text{Falsely rejecting } H_0 \mid H_0 \text{ is true})$

If $\alpha = 0.05$ then:

$$\text{Pr(}\text{not rejecting } H_0 \mid H_0 \text{ is true}) = 1 - 0.05 = 0.95$$
Multiple Comparisons

Type 1 error rate ($\alpha$): $Pr(\text{Falsely rejecting } H_0 \mid H_0 \text{ is true})$

If $\alpha = 0.05$ then:

$$Pr(\text{not rejecting } H_0 \mid H_0 \text{ is true}) = 1 - 0.05$$

$$= 0.95$$

However, there are 6 total unique comparisons that can be made on the same data. The probability of obtaining the correct decision in all comparisons made is:

$$(1 - \alpha)^6 = (1 - 0.05)^6$$

$$= 0.74$$

The type I error rate is inflated to:

$$1 - 0.74 = 0.26$$